

DP Analysis & Approaches (HL)

Maths with Lemon

HL Topic 4 – Bayes' Theorem Worksheet

Mathematician: _____

Warm Up

There are 21 students in a class. Each independently chooses a random integer from 1 to 100. Find the probability that at least two students choose the same number.

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A. Bayes' Theorem for Two Events

1. A bag contains 9 white and 6 black marbles. Two are drawn without replacement. Given that the colours are different, find the probability that the first marble was white.

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2. Two-thirds of a class are female. One-fifth of the girls study biology and half of the boys study biology. Find the probability that a student is female given that the student does not study biology.

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3. Sean wears a red shirt with probability $\frac{1}{5}$. Given a red shirt, he wears a pink tie with probability $\frac{3}{10}$; otherwise the probability is $\frac{7}{10}$. Given that he wears a pink tie, find the probability that he wears the red shirt.

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[illegible]

B. Three Events

4. Zoe's first chess match is a win, draw or loss with probabilities 0.6, 0.3, 0.1. Her probability of winning the tournament is 0.7, 0.4, 0.2, respectively. Given that she wins the tournament, find the probability that she drew her first match.

5. Jaxon travels by bus on Monday and Friday, taxi on Tuesday and Wednesday, and subway on Thursday. He is on time 90%, 99%, 95%, respectively. Given that he was late on a randomly selected working day, find the probability that he travelled by bus.

6. Waleed walks 60%, cycles 30%, and gets a ride 10% of the time. His late rates are 40%, 20%, 5%, respectively. Given that he is late, find the probability that he did not walk.

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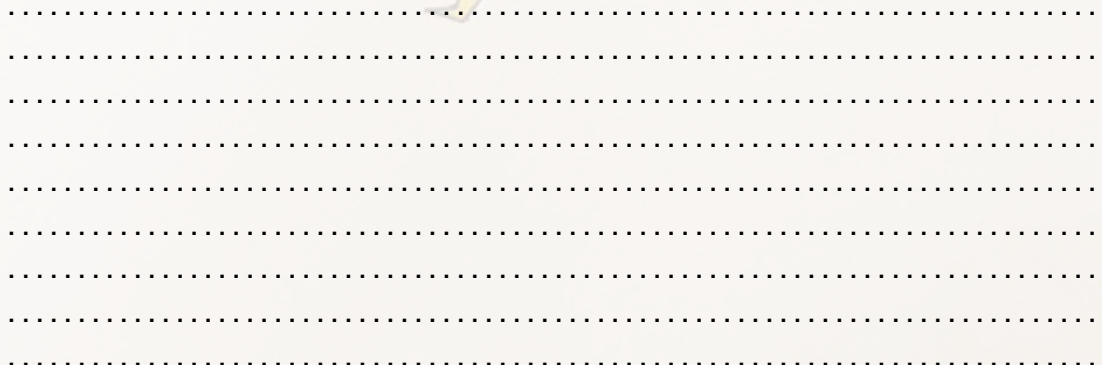
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A hand-drawn map of a coastal area. The land is represented by a yellow-shaded region with a black outline. The sea is represented by a blue hatched area to the left of the land. The map is drawn on a background of horizontal dotted lines.

8. Find $P(A' \mid B)$ if





10. Machines M_1, M_2, M_3 sort 30%, 30%, 40% of material and are not in use 3%, 7%, 5% of the day. A machine is not in use. Find the probability it is M_1 or M_2 .

11. Tram line U uses three times as many wagons as W. Late rates are $\frac{1}{10}$ on U and $\frac{1}{15}$ on W. A late tram is observed. Find the probability it is from W.

12. Of 500 cyclists, 50 use a banned substance. A test detects 45% of users but gives false positives for 8.5% of non-users. Given a positive result, find the probability the cyclist used the substance.

13. Box 1 has 3 red winning balls and 2 yellow non-winning balls. Box 2 has 2 red non-winning balls and 8 yellow winning balls. A box is chosen randomly and then a ball is chosen. Given that the ball is red, find the probability it is winning.



Analytical Solutions

Warm Up

Use the complement:

$$P(\text{match}) = 1 - P(\text{all different}) = 1 - \left(1 \cdot \frac{99}{100} \frac{98}{100} \cdots \frac{80}{100}\right).$$

Hence

$$P(\text{match}) \approx 0.891.$$

1. Marbles

Let D mean different colours.

$$P(W_1 \cap D) = \frac{9}{15} \frac{6}{14} = \frac{9}{35}.$$

$$P(D) = \frac{9}{15} \frac{6}{14} + \frac{6}{15} \frac{9}{14} = \frac{18}{35}.$$

Thus

$$P(W_1 | D) = \frac{9/35}{18/35} = \frac{1}{2}.$$

2. Biology

$$P(B') = \frac{2}{3} \frac{4}{5} + \frac{1}{3} \frac{1}{2} = \frac{7}{10}.$$

Therefore

$$P(F | B') = \frac{\frac{2}{3} \frac{4}{5}}{7/10} = \frac{16}{21}.$$

3. Shirt and Tie

$$P(T) = \frac{1}{5} \frac{3}{10} + \frac{4}{5} \frac{7}{10} = \frac{31}{50}.$$

Hence

$$P(R | T) = \frac{\frac{1}{5} \frac{3}{10}}{31/50} = \frac{3}{31} \approx 0.0968.$$

4. Zoe

$$P(T) = 0.6(0.7) + 0.3(0.4) + 0.1(0.2) = 0.56.$$

$$P(D | T) = \frac{0.3(0.4)}{0.56} = \frac{3}{14} \approx 0.214.$$

5. Jaxon

$$P(L) = \frac{2}{5}(0.10) + \frac{2}{5}(0.01) + \frac{1}{5}(0.05) = 0.054.$$

$$P(B | L) = \frac{\frac{2}{5}(0.10)}{0.054} = \frac{20}{27} \approx 0.741.$$

6. Waleed

$$P(L) = 0.6(0.4) + 0.3(0.2) + 0.1(0.05) = 0.305.$$

$$P(W' \cap L) = 0.3(0.2) + 0.1(0.05) = 0.065.$$

Thus

$$P(W' | L) = \frac{0.065}{0.305} = \frac{13}{61} \approx 0.213.$$

7. Airport

$$P(N) = 0.5(0.9) + 0.3(0.85) + 0.2(0.8) = 0.865.$$

$$P(H | N) = \frac{0.3(0.85)}{0.865} = \frac{51}{173} \approx 0.295.$$

8. Complement

$$P(B) = 0.3(0.2) + 0.7(0.4) = 0.34.$$

$$P(A' | B) = \frac{0.7(0.4)}{0.34} = \frac{14}{17} \approx 0.824.$$

9. Vroom

Let $x = P(F | C)$. Since $P(C) = 0.25$,

$$P(F) = 0.4(0.1) + 0.35(0.08) + 0.25x = 0.068 + 0.25x.$$

Given $P(C | F) = 0.15$,

$$0.15 = \frac{0.25x}{0.068 + 0.25x}.$$

Thus

$$0.0102 + 0.0375x = 0.25x, \quad x = 0.048.$$

Therefore

$$4.8\%.$$

10. Recycling Machines

$$P(N) = 0.3(0.03) + 0.3(0.07) + 0.4(0.05) = 0.05.$$

$$P(M_1 \cup M_2 | N) = \frac{0.3(0.03) + 0.3(0.07)}{0.05} = 0.6.$$

11. Tram Lines

Since U has three times as many wagons,

$$P(U) = \frac{3}{4}, \quad P(W) = \frac{1}{4}.$$

$$P(L) = \frac{3}{4} \frac{1}{10} + \frac{1}{4} \frac{1}{15} = \frac{11}{120}.$$

Therefore

$$P(W | L) = \frac{\frac{1}{4} \frac{1}{15}}{11/120} = \frac{2}{11}.$$

12. Banned Substance

$$P(D) = 0.1, \quad P(D') = 0.9.$$

$$P(+) = 0.1(0.45) + 0.9(0.085) = 0.1215.$$

Hence

$$P(D | +) = \frac{0.1(0.45)}{0.1215} = \frac{10}{27} \approx 0.370.$$

13. Two Boxes

A red winning ball can only come from Box 1:

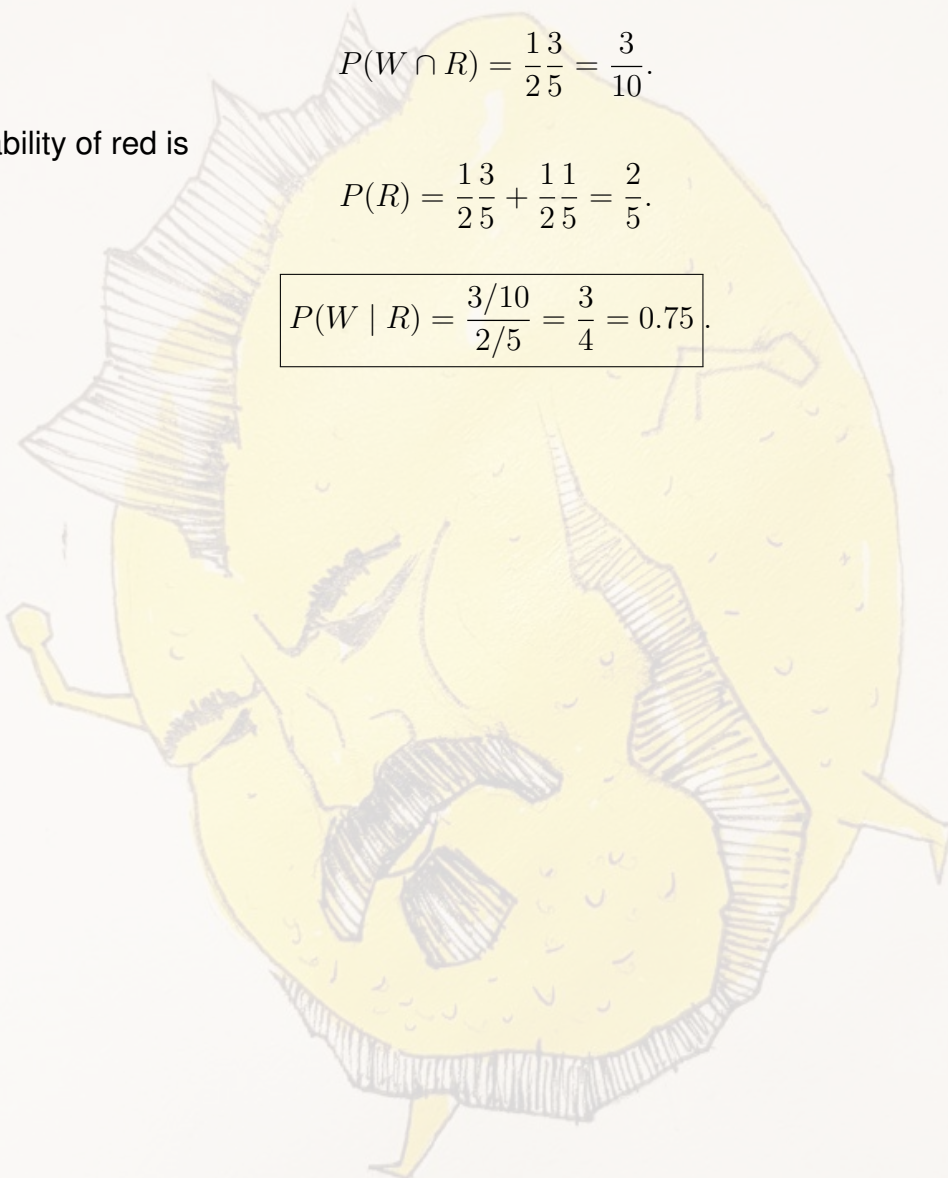
$$P(W \cap R) = \frac{1}{2} \cdot \frac{3}{5} = \frac{3}{10}.$$

The total probability of red is

$$P(R) = \frac{1}{2} \cdot \frac{3}{5} + \frac{1}{2} \cdot \frac{1}{5} = \frac{2}{5}.$$

Thus

$$P(W | R) = \frac{3/10}{2/5} = \frac{3}{4} = 0.75.$$





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Mathematics made clear.